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AN EXPERIMENTAL STUDY OF THE NATURAL CONVECTION FLOW OVER A HEATED RIDGE IN AIR

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Abstract—Measurements of natural convection transport to air above a heated symmetric ridge were made with a fine thermocouple probe and an interferometer. The ridge was formed of two inclined heated plates, hinged at their common trailing edge. Resulting data on the thermal structure of the flow is compared with that above isolated inclined surfaces. Wollaston prism schlieren interferometer results are correlated with the data from thermocouple traverses. The plume flow shed above the ridge was probed and interferograms were obtained. The plume is highly disturbed for flatter ridges. This is related to the more disturbed nature of the upstream flows at lower inclinations. The onset of upstream flow instabilities was also investigated. Past investigations with isolated surfaces classified these instabilities as either separation or transition, depending on the method of disturbance or transport detection. We found that disturbances have the characteristics of both separation and transition. Our determinations of the onset of instability, as judged by sudden growth of the thermal boundary region thickness, by changing heat-transfer characteristics and by flow visualization are discussed and compared with published data for isolated surfaces.

NOMENCLATURE

- A, B, constants used in the heat-transfer correlations;
- Gr_x, local Grashof number $= g\beta\Delta T x^3/\nu^2$;
- L, length of one side of the ridge;
- Nu_x, local Nusselt number $= h_x x/k$;
- Pr, Prandtl number $= \mu c/k$;
- Ra_x, local Rayleigh number $= Gr_x \cdot Pr$;
- $\bar{t}$, mean (time average) temperature in the boundary layer [°C];
- t_0, plate temperature [°C];
- t_∞, ambient temperature [°C];
- ΔT, $t_0 - t_\infty$ [°C];
- T', disturbance amplitude ratio = amplitude of disturbance/ΔT;
- x, distance from the leading edge of the plate [cm];
- y, distance normal to the plate [cm].

Greek symbols

- α, angle measured from the vertical [deg];
- δ, local thickness of the boundary layer [cm];
- θ, angle measured from the horizontal [deg];
- η, similarity variable for horizontal laminar flow; $= (y/x)(Gr_x/5)^{0.2}$;
- $\bar{\phi}$, nondimensional mean temperature; $= (\bar{t} - t_\infty)/(t_0 - t_\infty)$.

INTRODUCTION

THE CHARACTERISTICS of natural convection flow of air over a heated ridge were measured at slopes to 30° from horizontal. Flows form separately on each side and interact along the ridge, in a complicated manner, to form a plane plume. These flows have been little studied at small scale in spite of their importance. They affect transport and cause micro-meterological patterns such as thermal and water vapor transport. Analysis of such transport would be extremely complicated. The first study was experimental, to first clarify some of the detailed mechanisms.

Isolated inclined surfaces have been studied in the past. At high inclination the transport is similar to that in vertical flows, even though a normal component of buoyancy force exists. In the extreme case, of a horizontal orientation, that is the only component. Vertical laminar transport is quite well understood. In the boundary-layer regime such flows eventually undergo transition to turbulence. Early work concerning horizontal and inclined flows indicated another mechanism of instability, which has been called separation. It results from the normal component of buoyancy. Hassan and Mohamed [1] have suggested that it occurs much earlier than turbulence. Pera and Gebhart [2] noted that separation in natural convection is driven by the buoyancy force instead of by an external pressure gradient, as in forced flows. This very much complicates any "separation" mechanisms which may arise. In other studies of flows inclined slightly from the horizontal, both terms, transition and separation, have been used. Here the meaning of the terms transition and separation are investigated and somewhat clarified.

Schmidt [3] apparently first studied natural convection boundary layers over an inclined plate. A schlieren system was used. Rich [4] used a Mach–Zehnder interferometer to measure temperature profiles and to obtain the heat transfer from a plate inclined up to 40° from the vertical. The heat-transfer results were correlated in the form $Nu = f(Pr \cdot Gr \cos\alpha)$, where α is the angle of inclination

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from the vertical, and the Grashof number does not include any dependence on angle. Kierkus [5] used a perturbation analysis about the vertical to predict velocity and temperature profiles and heat transfer for isothermal surfaces inclined up to 45° from the vertical. Experiments performed in air agreed well with the analysis.

Hassan and Mohamed [1] measured local heat-transfer coefficients in air along an isothermal flat plate, inclined from +90 to −90° from the vertical in increments of 15°. Hot and cold surfaces facing up and down were investigated. The heat flux was measured using Boelter–Schmidt type heat flux meters. The measurements were correlated in terms of local Nusselt number as a function of the local Grashof number. The Grashof number was based on the component of the buoyancy force parallel to the surface. Their correlations all assumed $Nu_x = A(Pr, Gr_x, \cos\alpha)^{1/4}$ in the laminar region, where the angle dependence has been separated from the Grashof number. For hot surfaces facing upwards, it was found that the heat flux along the plate decreases to a minimum, increases and then remains almost constant. The location of the minimum was taken to indicate flow separation and occurred well before expected turbulence. Higher surface temperatures with respect to the ambient and larger angles from the vertical were found to cause earlier minimums. No visual studies were made.

Fujii and Imura [6] measured heat transfer and used a shadowgraph to visualize the flow over inclined plates in water. The surface condition was neither isothermal nor uniform flux. From their correlations of Nusselt number vs Rayleigh number, they defined separation from a heated surface facing upward. The deviation of the heat-transfer data from the laminar trend was referred to as both separation and transition. Results were compared to Vliet [7] and were found to be included within Vliet's beginning-end of transition regime. The data of Lloyd and Sparrow [8] was also compared and found to be about an order of magnitude lower.

Pera and Gebhart [2] performed a perturbation analysis for boundary-layer flow over a flat plate inclined at small angles to the horizontal. The Grashof number was modified by the cosine of the angle from the horizontal. This is the normal component of buoyancy and provides the indirect drive mechanism. Temperature profiles were measured in air with a Mach–Zehnder interferometer and used to obtain heat transfer. The analysis predicted increasing heat transfer with increasing angle. The experimental results confirmed this but also showed increased heat transfer for $Gr_x > 10^4$. Flow separation was visualized by smoke injection. Separation was found to occur for $Gr_x^{1/3} > 140$. Artificial disturbances at the leading edge of the flow caused separation to occur earlier.

A subsequent study by Pera and Gebhart [9] investigated the stability of boundary-layer flows over an inclined plate, near horizontal. Further flow visualizations with smoke were performed on an inclined plate with leading and trailing adiabatic extensions and also on a ridge to investigate the effect on the laminar flow and the beginning of flow separation. No detectable differences were found.

Jones [10] used series solutions, one valid near the leading edge, another further downstream, to solve the flow over a flat plate inclined at small angles with the horizontal. The two different solutions result from the consideration of the driving mechanism. In the vicinity of the leading edge the tangential force has an insignificant effect. The flow is driven indirectly by the motion pressure gradient resulting from the normal component. At large distances downstream, the fluid layer thickens sufficiently enough to generate tangential buoyancy forces which dominate the pressure gradient driving forces. No experiment was performed to verify the analysis.

Black and Norris [11] performed experiments in air over inclined surfaces. The boundary-layer flow was visualized with a differential interferometer which allowed direct measurement of heat flux in the laminar, transition and turbulent regimes. They also reported data on the frequency of occurrence of thermal waves.

Several other studies have dealt with inclined plates and the boundary-layer flow instabilities which result. Most have considered plates inclined at small angles with the vertical. Shaukatullah [12] reviews these and provides additional data on the instability mechanisms occuring in near vertical flows. Simultaneous velocity and temperature measurements in the boundary layer were used to show the existence, size, and growth of a longitudinal vortex system over a uniform flux surface in water.

The present study extends the understanding of flow over inclined surfaces, providing more detailed data on the thermal structure in the laminar region and beyond. These results, on a ridge, relate the single inclined plate to a more complicated geometry. Visual results from a Wollaston prism schlieren interferometer designed by Jahn [13], are correlated with those of a fine wire thermocouple. Some comments on the interaction of the two boundary layers as they join are also included. Pera and Gebhart [9] observed that, for the ridge shaped geometry, the separated flow from each side apparently met and rose in a single plume. No plume measurements were made. Eroshenko *et al.* [14], in describing and demonstrating a modified Mach–Zehnder interferometer, presented interferograms of a ridge shaped geometry. The manner in which the ridge was heated was not described. No conclusions about the flow were presented

EXPERIMENTAL APPARATUS AND PROCEDURE

Our experiment was performed over a ridge made up of two aluminum plates hinged at a common edge. They were insulated on the sides and bottom. Each plate was 35.6 cm wide and 24.1 cm long and had six strip heaters embedded spanwise under the plate. Over each heater was a copper–constantan thermocouple 0.03 cm below the surface. Individual rheostats in the heater circuits were adjusted to attain an isothermal surface condition with 0.5% or less variation in ΔT along each side and a temperature difference between the two sides of less than 1%. Details on the construction of the plates are contained in Pera and Gebhart [2, 9]. The plates were set at various inclinations, which were measured with a machinist's inclinometer to ± 2 min of a degree.

A box, of polyethylene sheet on a wooden skeleton, was built to house the experiment and isolate it from the environment. The box was 2.13 m high and of 0.22 m^2 cross-section. Because of the heavy construction of the plate, steady state was reached after about 1 h. Therefore, it was necessary to allow air flow through the box to prevent stratification. Thermocouples located in the top and bottom of the box were monitored and stratification was always less than 1% of the plate ΔT. The top of the box and a strip around the bottom were provided with layers of cheesecloth to damp any disturbances which might be present. The sides of the box were provided with optical quality glass windows to allow for flow visualization. Side walls were also constructed for the plate to insure a 2-dimensional flow geometry.

Flow visualization was accomplished with a Wollaston prism schlieren interferometer. Sernas and Fletcher [15] and Black and Carr [16] have utilized this technique and compared measured results with analyses. Jahn [13] has investigated the principles of this interferometer in detail. Significant contradictions in the method of heat-transfer measurement were found in previous studies and were corrected.

Preliminary experiments showed the Wollaston interferograms to be inconclusive in defining flow separation. Disturbances were seen over a wide range of downstream locations and were convected downstream. Due to spanwise averaging of the interferometer, information on 3-dimensional disturbances was obscured. Therefore, five chromel–alumel thermocouples, made from 0.00254 cm (0.001 in) wire, were assembled into a probe to measure temperature disturbances in the boundary layer. Simultaneous recording of any four thermocouples signals and observation of the interferometer results showed the disturbances to be random in space and time across the plate. Their scale was less than or equal to the probe spacing, which was 2.54 cm. The final measurement of temperature profiles across the boundary layer was done with a chromel–alumel thermocouple made from 0.00127 cm (0.0005 in) wire. Details of probe construction and thermocouple signal recording are contained in Miller [17].

The procedure for each run was to adjust the temperature uniformly along the plate and at the desired level. Boundary-layer traverses were made in the center of the span of the plate at 8–10 different downstream locations for each run. Approximately 20 temperature measurements were made across the boundary layer at each downstream location.

From the average thermocouple signal recordings, mean temperature profiles were calculated. A linear least square fit was used near the wall to obtain the slope, thus giving the local heat flux. It was not possible to obtain heat-transfer measurements from the electrical input due to unknown losses through the insulation and an uncertain radiation correction. A least square polynomial fit of fourth order was used to define the mean boundary region edge.

The coefficient of thermal expansion was evaluated at the ambient temperature. Other fluid properties were evaluated at a reference temperature defined as 62% of the plate ΔT above the ambient temperature, as suggested by Sparrow and Gregg [18] for vertical flows.

RESULTS AND DISCUSSION

Temperature measurements.

(A) *Mean temperature profiles.* The accuracy of the experiment and instrumentation was assessed by comparing measured temperature profiles at various downstream locations with the calculated profile of Pera and Gebhart [2], for both surfaces horizontal. For small distances downstream the profiles agree well, especially near the wall where the heat-transfer data is taken, see Fig. 1. The profile for $Gr_x = 1 \times 10^5$, where x

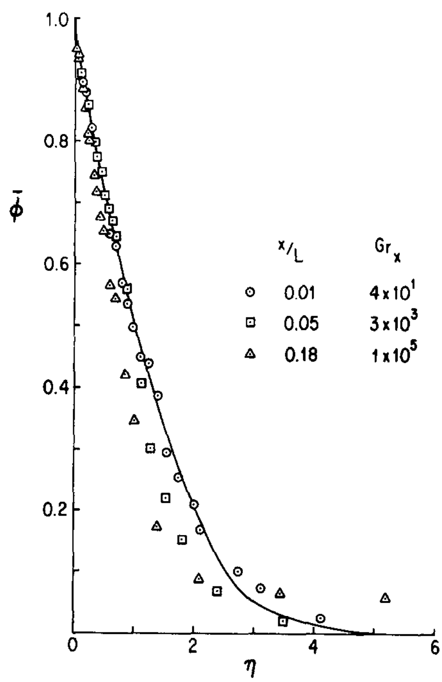

FIG. 1. Mean temperature profiles over a horizontal surface, $\Delta T = 11.7$°C, —— theoretical profile, Pera and Gebhart [2].

= 4.3 cm (1.7 in) already shows steepening near the wall and thickening in the outer region. The increased slope at the wall results in increased heat transfer, in agreement with the results of Pera and Gebhart [2].

A similarity solution does not exist, in general, for inclined plates. Therefore, the mean temperature profiles were reduced in terms of the physical distance normal to the plate. Figure 2 shows the thermal boundary region thickness in centimeters, vs downstream distance x/L ($L = 24.1$ cm). The y locations of the mean isotherms in the boundary region, at 5, 10,

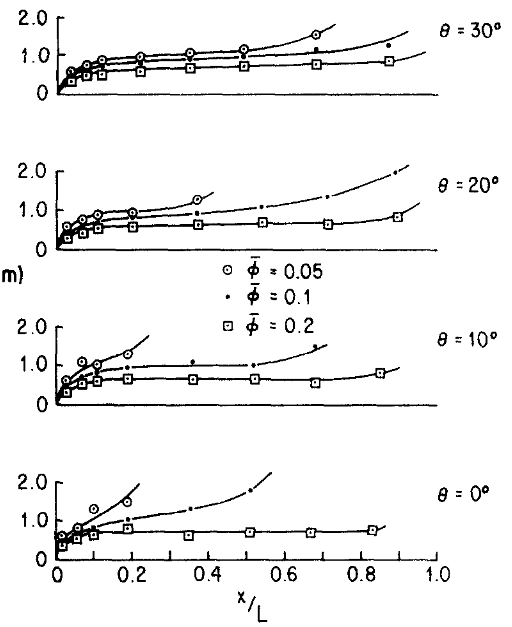

FIG. 2. Effect of inclination on mean boundary region growth, $\Delta T = 27.8°C$, in terms of mean temperature distribution $\bar{\phi}$.

and 20% of the plate ΔT above the ambient temperature, are shown. The 5% isotherm is taken to represent the edge of the boundary region. The rapid thickening indicates hot fluid escaping from the boundary region. Visual observations confirmed earlier findings by Fujii and Imura [6] that these elements of hot fluid do not necessarily rise vertically, but are instead convected along outside the boundary region. The 20% isotherm location is seen to be approximately independent of angle and never separates, in the mean sense, until the two boundary layers interact and are forced to begin to rise, at $x/L = 0.9$.

(B) *Temperature fluctuations.* Increasing ΔT increases the frequency of the occurrence of disturbances but does not change the average first downstream location of their appearance. When a disturbance became noticeable it was seen to be convected downstream. Another disturbance would then appear behind the first. The point of apparent origin of disturbances moved about with time. The range of motion was as much as 50% of the total length of a side of the ridge. Characteristic infinite fringe interferograms are seen in Fig. 3. The effect of increasing inclination in stabilizing the flow is very apparent. At higher angles the disturbances are smaller and remain closer to the surface. They blend together to form an outer layer.

The unfiltered analogue thermocouple signals were analyzed for the amplitude of the temperature disturbances in the boundary region. The downstream variation of T', defined as the ratio of disturbance amplitude to the total ΔT, is plotted in Fig. 4, for $\theta = 0$ and 30°. These amplitudes are determined at the 50% mean isotherm, the temperature center of the boundary region. That is, at $\bar{\phi} = (\bar{t} - t_\infty)/(t_0 - t_\infty) = 0.5$. Surprisingly, we see that the imposed ΔT level has no progressive effect on T'. Disturbance amplitude is seen first to grow rapidly downstream and then become constant. A least square polynomial fit of the data, at each inclination was also determined, but is not shown.

Distributions of the temperature disturbance T' across the thermal boundary region, plotted against y/δ where δ is taken as the thickness at $\bar{\phi} = 0.2$, are shown in Fig. 5 for $\theta = 10°$. The 20% mean isotherm was seen to represent normal boundary region growth, since separation effects have not changed the mean profile there. Disturbance amplitude increases with inclination. A very similar effect for constant angle and increasing x/L was found, as seen in Fig. 5.

Figure 6 compares the thermocouple analog output of T' with a corresponding interferogram. We see that an interferogram, which is a spanwise average, may be very deceiving. The probe location is indicated in the interferogram. The probe shows temperature disturbances for which $T' = 0.4$, when nothing is visible in the interferogram.

(C) *Temperature measurements in the plume.* The shed flow region was scanned for $\Delta T = 1.8$, 3.1 and 5.8°C at $\theta = 30°$. Traverses were made vertically above the apex of the ridge and also horizontally across the region above the apex. The analog results indicated that the plume was somewhat like that from a line source. Two characteristic disturbances were seen in the analog record. One was of high amplitude and short duration and was similar to those observed in the ridge boundary region. These were thought to be those convected along in the attached boundary region flow. The other type were of large amplitude and long period. Bill and Gebhart [19] report similar disturbances in the plume above a heated wire. Simultaneous flow visualization with a Mach–Zehnder interferometer showed these disturbances to be due to the swaying of the plume normal to the plane of symmetry. They considered the maximum measured temperature as representative of the centerline temperature. Our plume was far more complicated, because of interaction with the many disturbances shed with the boundary region flows. There was no characteristic maximum temperature. We might expect that higher inclination, which might result in a more stable boundary region flow, would result in a more ordered shed plume. See also, as other indications of the properties of shed flows, the visualizations of Pera and Gebhart [20] on an inverted U geometry and the detailed measurements of Jaluria and Gebhart [21] above a hemisphere with the base horizontal.

An interferogram of the shed plume above the ridge is seen in Fig. 7. The center fringes show the merging interactions, which rise in a single plume. The plume sways, which was seen in the large amplitude and long period disturbances detected by the thermocouple probe. On the right side of the ridge are two large disturbances, seen as sharp bends in the fringes. By relating similar disturbances to the thermocouple analog output, we determined that these disturbances

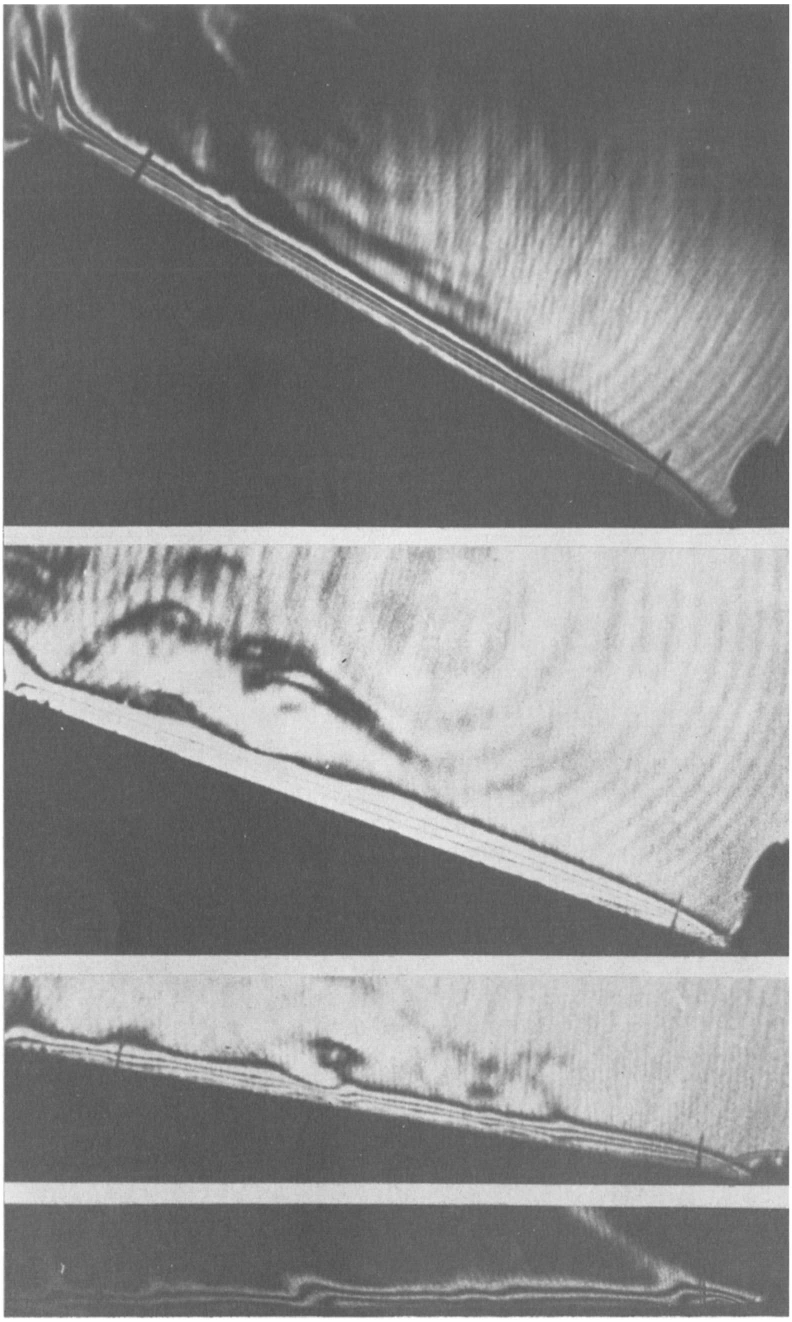

FIG. 3. The effect of inclination on boundary region flow, for 0, 10, 20, 30° inclination. $\Delta T = 27.8°C$.

were the source of the short duration, high amplitude disturbances in the plume. They were found to be very random.

Nevertheless, the shed flow was contained in a relatively well defined plume. This is in accord with our earlier observation that, for an inclination of 30°, the disturbances do not leave the boundary region. They are convected along it and entrained back into the boundary region.

(D) *Heat transfer*. A linear least squares fit was performed on the mean temperature data near the surface. Approximately ten data points were available in the approximately linear region of each profile. The slope of the fit at the surface was then used to compute the local heat flux, in terms of Nusselt number, $Nu_x = h_x x/h$. Figures 8 and 9 present typical results vs local Rayleigh number, $Ra_x = (gx^3\beta\Delta T/\nu^2)Pr$, for inclinations of 0° and 30°. All of the data followed a linear trend in the lower range of Ra_x. However, it diverged upwards in the range of $Ra_x = 10^6 - 10^7$, depending on angle. This effect is much less pronounced at 30°. As in the results of Fujii and Imura [6], the divergence was taken to indicate a significant change in flow behavior.

The data before divergence is here referred to as the laminar trend, despite the presence of high disturbance

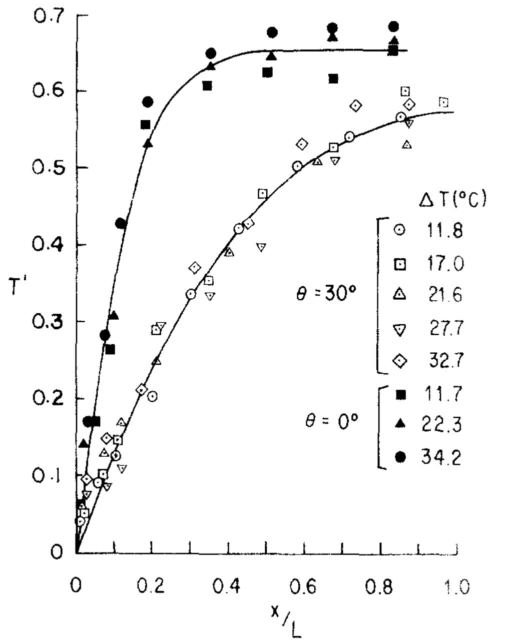

FIG. 4. Temperature disturbance amplitude growth T', along the 50% mean isotherm ($\phi = 0.5$) for inclinations of 0 and 30°.

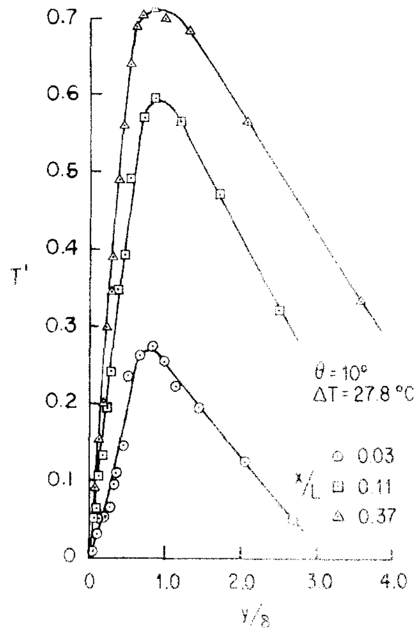

FIG. 5. The effects of downstream location, x/L, on the distribution of temperature disturbances across the boundary region.

levels. Heat transfer in this region was correlated in the form $Nu_x = ARa_x^B$ for each angle. The results are collected and compared with other results for inclined surfaces in Table 1 and also in Fig. 10. The intercept A and exponent B both depend on inclination. For $\theta = 0°$, analysis indicates $B = 0.2$, see for example, Pera and Gebhart [2]. The best fit of the present experimental data for $\theta = 0°$ gives $B = 0.227$. Lowering the upper limit of local Rayleigh number below those listed in Table 1 has little effect on the fits. Higher heat-transfer rates were also found by Pera and Gebhart [2], for a local Grashof number greater than 10^4. This is prob-

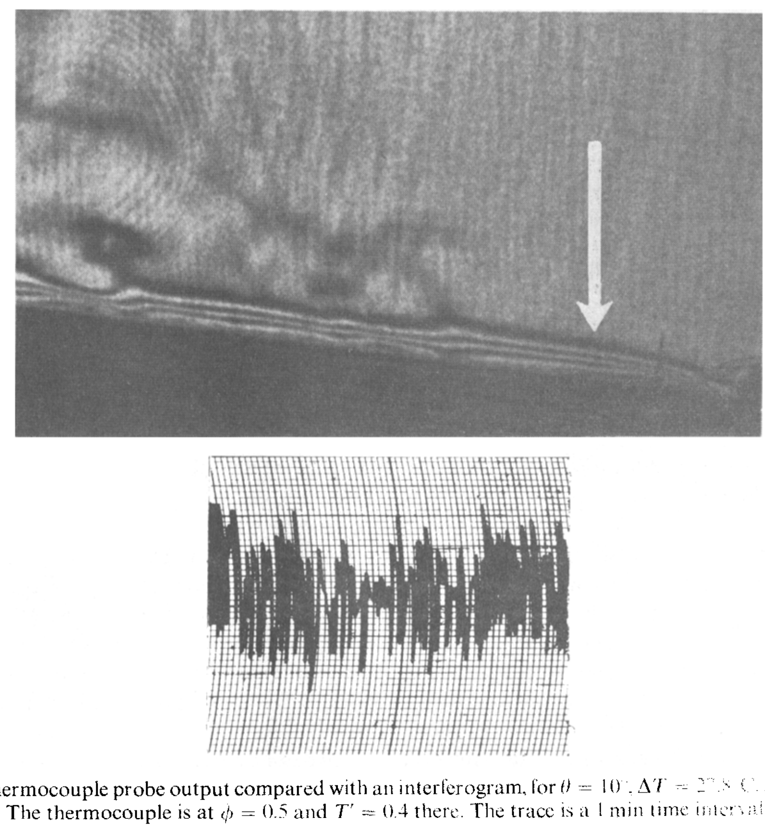

FIG. 6. Thermocouple probe output compared with an interferogram, for $\theta = 10°$, $\Delta T = 27.8°C$, $x/L = 0.17$. The thermocouple is at $\phi = 0.5$ and $T' = 0.4$ there. The trace is a 1 min time interval

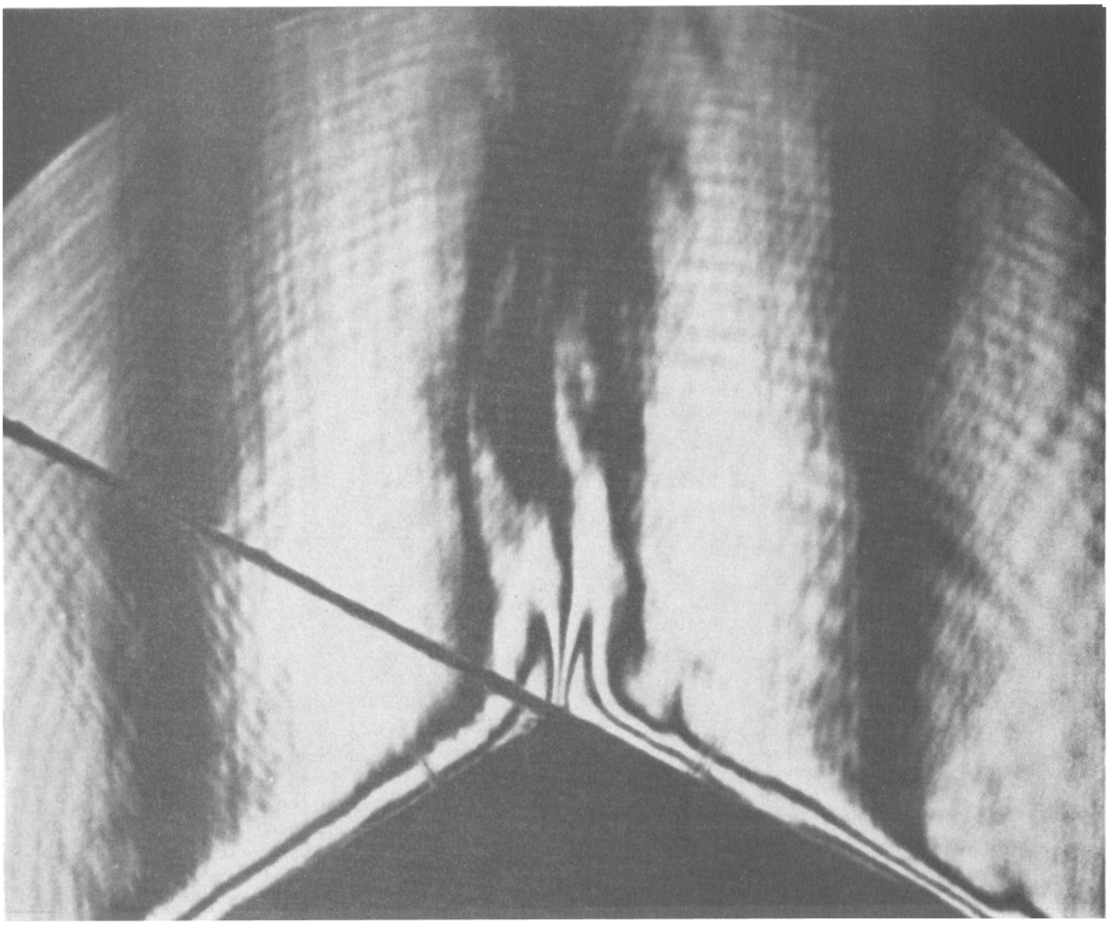

FIG. 7. Interferogram of the flow shed from the ridge. $\theta = 30°$. $\Delta T = 27.8°C$.

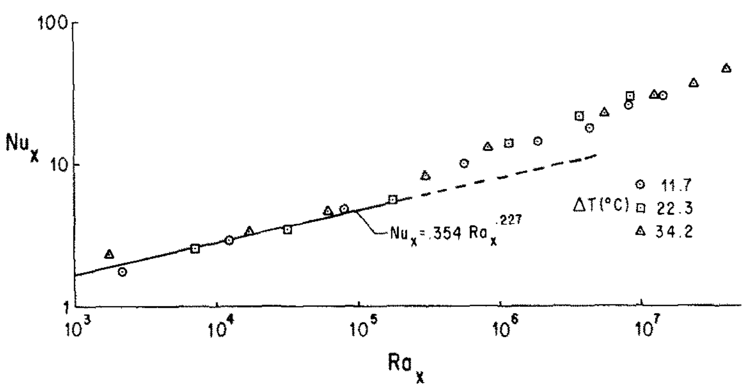

FIG. 8. Local heat-transfer data for $\theta = 0°$ at various values of ΔT. The laminar trend correlation found here is also plotted.

ably due to the short attached flow region and early appearance of disturbances, for inclinations near horizontal.

The cosine of the angle of inclination was initially included in the local Grashof number, since the analysis of Pera and Gebhart [2] showed this to be the appropriate component of buoyancy for flows at small angles from the horizontal. However, it did not collapse the low Rayleigh number data at various inclinations. Hassan and Mohamed [1] found that the tangential component of buoyancy, as indicated by the cosine of the angle from the vertical, correlated heat-transfer results very well for plates inclined more than 30° from the horizontal. However, their data at 30° showed a trend away from this behavior, and data at 15° had to be correlated separately. The present correlations thus do not include any angular dependence in the Grashof or Rayleigh numbers. The value of the exponent, B, increases with increasing inclination. For 30° it is essentially that for vertical surfaces, 0.25, and that value was used.

Our data above the Rayleigh number limits for the

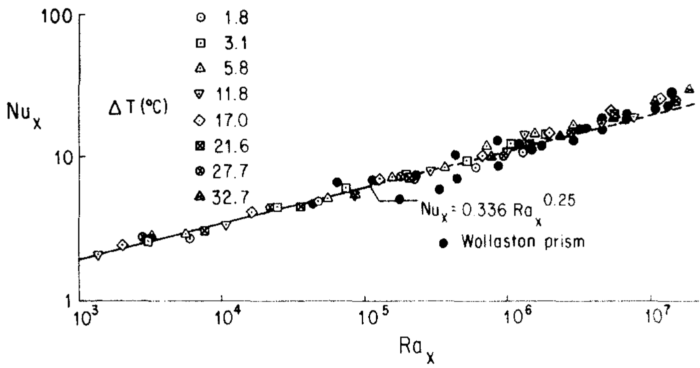

FIG. 9. Heat-transfer data for $\theta = 30°$, along with our laminar correlation. Solid symbols are from Wollaston prism schlieren interferograms.

Table 1. Heat-transfer correlations

Angle (deg)	$Nu_x = ARa_x^B$	Range of data (Ra_x)	Source
0	$0.354Ra_x^{0.227}$	$1 \times 10^3 - 2 \times 10^5$	Present results
10	$0.330Ra_x^{0.237}$	$2 \times 10^3 - 1 \times 10^5$	
20	$0.321Ra_x^{0.245}$	$2 \times 10^3 - 3 \times 10^5$	
30	$0.336Ra_x^{0.25}$	$2 \times 10^3 - 1 \times 10^6$	
15	$0.315Ra_x^{0.25}$	$1 \times 10^3 - 1 \times 10^6$	Hassan and
30	$0.319Ra_x^{0.25}$	$2 \times 10^3 - 1 \times 10^6$	Mohamed [1]
0	$0.383Ra_x^{0.2}$		Laminar theory for a horizontal isothermal surface, Pera and Gebhart [2]
90	$0.378Ra_x^{0.25}$		Laminar theory for vertical isothermal surface
0	$0.119Ra_x^{1/3}$	$3 \times 10^5 - 3 \times 10^7$	Present results in separated regimes
10	$0.119Ra_x^{1/3}$	$6 \times 10^5 - 2 \times 10^7$	
20	$0.119Ra_x^{1/3}$	$4 \times 10^6 - 4 \times 10^7$	

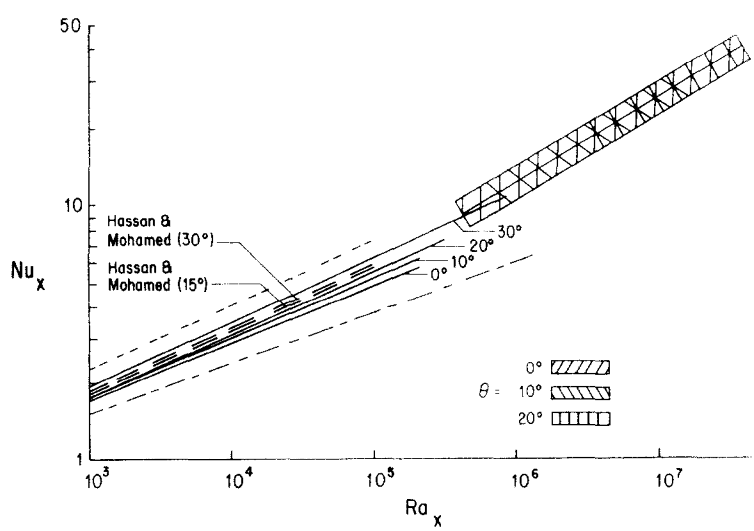

FIG. 10. Comparison of heat-transfer data with some published results. – – – present laminar trend correlations. ··· — vertical and ––––– horizontal boundary-layer calculations, Gebhart [25] and Pera and Gebhart [2]. — — Hassan and Mohamed [1], ▭ developed region heat transfer.

laminar trend agrees well with a single correlation for inclinations of 0, 10 and 20°. The flow appears to have suffered a kind of transition and to have become developed, in that there was no further dependence on x, i.e. $B = 1/3$. The constant A is also given in Table 1. The data at 30° lies slightly below that at lower inclinations and closer to the laminar trend as seen in Fig. 9. This is probably due to the increased thermal stability of this flow. It is still in a transition regime.

Our correlations, as well as other published results, are summarized in Fig. 10. All are bounded by the trends for horizontal and vertical orientations. The results of Hassan and Mohamed [1] are lower than the present results for 30° and higher for 15°. The difference is small however and may be partly due to their adherence to an exponent of $B = 0.25$. Also, our data is for a ridge. The interaction of the two sides may also have had an effect.

Jahn [13], has corrected the formulation for measuring heat transfer with a Wollaston prism schlieren interferometer. Such computed heat transfer results are also plotted in Fig. 9. They agree well with those from the measured profiles. However, there is considerably more scatter. This is due primarily to flow fluctuations which caused the fringes to shift slightly when a large disturbance passed the fringe location.

CONCLUSIONS

In the past, investigators have described the instability phenomena in natural convection boundary layers over nearly horizontal surfaces as either separation (Pera and Gebhart [9] and Hassan and Mohamed [1]) or as transition to turbulence (Fujii and Imura [6] and Black and Norris [11]). The present study has shown that the flow beyond the so-called laminar regime has characteristics suggesting that both phenomena are occurring simultaneously.

Flow separation occurs in the sense that some elements of warm fluid leave the boundary region. This separation is largely limited to the outer portion of the boundary region, however, since the 20% mean isotherms remain very near the surface. At inclinations near the horizontal, the escaping fluid rises nearly vertically. As the inclination increases, these elements of warm fluid are convected along the surface and are even entrained back into the boundary region. This characteristic was detected by Pera and Gebhart [9] in their smoke visualizations which showed the 3-dimensional nature of the flow. The smoke showed what appeared to be rapidly developing longitudinal rolls. This outer region was also detected by Black and Norris [11] who referred to it as transitional flow.

The rate of heat transfer has been found to behave in a manner characteristic of transport in a transitional flow by virtually all who have made measurements. That is, the heat transfer follows a trend as predicted by laminar boundary-layer theory and then increases and becomes independent of downstream location.

The downstream growth rate of temperature disturbances was found to be exponential and then to

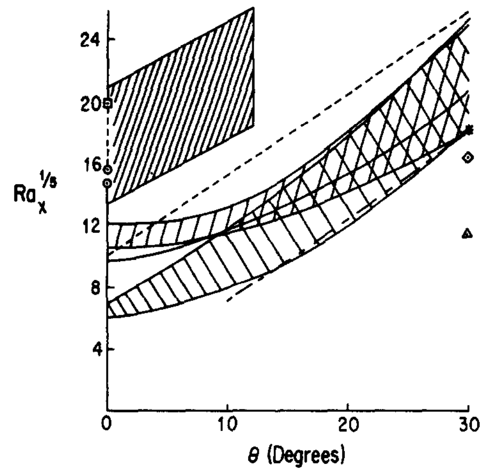

FIG. 11. Onset of flow instabilities vs angle of inclination from the horizontal. —— Average location of disturbances initially leaving the boundary region, as detected by interferograms (present results). ▧ Mean boundary-layer growth (present results). ▨ Deviation of heat transfer from laminar trends (present results). ▥ Visualization by smoke injection (Pera and Gebhart [9]). ------- Deviation of heat transfer from laminar trend (Fujii and Imura [6]). — — — Differential interferometer flow visualization (Black and Norris [11]). ○ 53.3 cm plate cellular instabilities (Rotem and Claassen [22]). □ 30.5 cm plate cellular instabilities (Rotem and Claassen [22]). △ Flow visualization (Lock, Gort and Pond [24]). ◇ Plate temperature variation (Vliet and Ross [23]). * Minimum heat-transfer rate (Hassan and Mohamed [1]).

approach nearly zero. Similar trends were found by Shaukatullah [12] in flows in water, inclined up to 30° from the vertical.

The major difference in reporting these phenomena has been in the criteria used to evaluate their effects. Figure 11 summarizes results of past investigators, as well as the findings of the present study. The coordinates are $Ra_x^{1/5}$ vs angle of inclination from the horizontal. The present study used the following criteria: the beginning of rapid growth of the boundary region, in terms of mean temperature isotherms, recall Fig. 2; the deviation of the transport rate from the laminar trend; and the average downstream location where fluid was first seen to leave the boundary region in the infinite fringe interferograms.

The present data, by all three criteria, is seen to be bounded by the results of Fujii and Imura [3] above, and by those of Black and Norris [11] below. From the trends, we see, for small inclinations, that the rapid growth of boundary region thickness occurs before the heat transfer deviates from its laminar trend. As the inclination increases, both effects occur at approximately the same downstream location. This may be partially due to the disturbances leaving the boundary region at small inclinations, and being re-entrained at larger inclinations. Therefore, at small inclinations, the inner region is not as affected by the disturbances as is the outer region.

Both the data of Rotem and Claasen [22] at $\theta = 0°$, and of Pera and Gebhart [9] for $\theta = 0$–$12°$, show the onset of instabilities to occur at higher Rayleigh numbers. Their criteria was, "the onset of cellular convection", and apparently relates to a further developed state. The criterion of Hassan and Mohamed [1] for separation was the occurrence of a minimum local heat-transfer rate. Only one of their points falls in the range of the present data. However, their data, which includes angles near the vertical, shows a smooth trend towards values typical of flows in transition for vertical surfaces. The single data point of Vliet and Foss [23] arises from a similar criterion. That of Lock, Gort and Pond [24] was determined by visualization.

In vertical flows it is hydrodynamic instability which eventually causes transition to turbulence. For horizontal flows, a thermal instability, as in the classical Benard problem, also arises and is the more probable effect which destroys the attached laminar flow. For inclinations in between it may be variable combinations of both modes. Our results, at these inclinations, suggest that separation and transition are one and the same, as far as most characteristics of the flow are concerned. In nearly horizontal flows, neither term, transition nor separation, with their definitions, apply. Transition is well established in vertical flows. There is clearly a change-over of mechanisms, with increasing inclination from the vertical. Surprisingly, the present data shows greater uncertainty of the location of events for higher inclinations from the horizontal, as indicated by the wider bands in Fig. 11. This suggests that the two modes of instability may be becoming comparable.

This data, on the onset of flow instabilities, provides a better understanding of the fluid mechanics of near horizontal flows. It emphasizes the need to be specific in defining criteria used to detect changes in flow behavior. Depending on its use, one criterion may be more applicable than another. What is clearly needed now is a detailed study of local flow quantities to determine the physical nature and behavior of the disturbances. This will be a very difficult study.

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ETUDE EXPERIMENTALE DE LA CONVECTION NATURELLE SUR UNE CRETE DANS L'AIR

Résumé—Des mesures sur la convection naturelle de l'air sur une crête symétrique chauffée ont été faites à l'aide d'une fine sonde à thermocouple et d'un interféromètre. La crête est formée par deux plaques inclinées jointes à leur bord de fuite. Les résultats sur la structure thermique de l'écoulement sont comparés à ceux relatifs à une surface inclinée. Les résultats donnés par l'interféromètre à prisme de Wollaston sont reliés à ceux donnés par le thermocouple dans le panache au dessus de la crête. La panache est fortement perturbé pour les crêtes les plus plates. Ceci est lié à la nature plus troublée des écoulements ascendants aux faibles inclinaisons. On étudie aussi l'apparition des instabilités de l'écoulement ascendant. Des études antérieures sur des surfaces uniques ont classé ces instabilités en séparation ou en transition selon la méthode de perturbation ou de détection du transport. On trouve que les perturbations ont à la fois les caractéristiques de la séparation et de la transition. Les déterminations de la mise en place de l'instabilité jugée sur la croissance soudaine de l'épaisseur de la zone thermique, sur le changement des caractéristiques du transfert thermique et sur la visualisation de l'écoulement, sont discutées et comparées aux résultats publiés pour les surfaces isolées.

EXPERIMENTELLE UNTERSUCHUNG DER FREIEN KONVEKTION VON LUFT ÜBER EINEM BEHEIZTEN PRISMA

Zusammenfassung—Es wurden Messungen bei freier Konvektion von Luft über einem beheizten, symmetrischen Prisma mit einem dünnen Thermoelementfühler und einem Interferometer vorgenommen. Das Prisma wurde aus zwei geneigten beheizten Platten gebildet, die gelenkig an der gemeinsamen Abströmkante verbunden waren. Die gefundene thermische Strömungsstruktur wurde mit Ergebnissen verglichen, die an einzelnen geneigten Platten gewonnen wurden. Messungen mit einem Wollaston-Prismen-Schliereninterferometer wurden mit den Thermoelementmessungen in Beziehung gebracht. Die Strömung über dem Prisma wurde untersucht und in Interferogrammen dargestellt. Die Strömung ist umso turbulenter, je stumpfer das Prisma ist. Des weiteren das Einsetzen von Strömungsinstabilitäten untersucht. In früheren Untersuchungen mit Einzelflächen wurden diese Instabilitäten entweder als Ablösung oder als Übergang bezeichnet, je nach der Art der Störung oder der Beobachtung. Wir fanden heraus, daß die Störungen sowohl Ablösungen als auch Übergänge sein können. Das plötzliche Anwachen der thermischen Grenzschicht halten wir für die Ursache des Einsetzens der Instabilität, unsere Untersuchungen darüber werden verglichen mit veröffentlichten Ergebnissen, die für einzelne Oberflächen gelten.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В ВОЗДУХЕ НАД НАГРЕТОЙ КРОМКОЙ

Аннотация — С помощью микротермопары и интерферометра проводились исследования естественной конвекции воздуха над нагретой кромкой. Кромка образована двумя наклоненными и шарнирно соединенными в месте контакта нагретыми пластинами. Полученные результаты по распределению температур в потоке сравниваются с результатами для отдельных наклонных поверхностей. Интерферометрические результаты измерений совпадают с результатами измерения термопарой. Отмечено очень сильное возмущение восходящего потока при плоской геометрии кромки, что объясняется большим искажением вертикальных потоков при малых углах наклона. Также исследовалось возникновение неустойчивости вертикального конвективного движения. В проводимых в прошлом исследованиях на обособленных поверхностях эти неустойчивости классифицировались или же как отрыв потока или переход в зависимости от метода внесения возмущения или способа регистрации. В данной работе найдено, что возмущения имеют характеристики как отрыва, так и перехода. Результаты по возникновению неустойчивости, подтверждаемой мгновенным увеличением толщины теплового пограничного слоя, изменением интенсивности теплообмена и визуализацией потока, обсуждаются и сравниваются с опубликованными данными для отдельных поверхностей.